Computational PDEs

1.1 Course Number: MA 321

1.2 Contact Hours: 3-0-0 Credits: 9

1.3 Semester-offered: 3rd Year-Odd

1.4 Prerequisite: Linear Algebra; Calculus; Differential Equations; Numerical Methods and Matlab

1.5 Syllabus Committee Member: Dr. C. Kundu, Dr. M.K. Rajpoot (convener), Dr. A. Kumar, Dr. G. Rakshit.

2. **Objective:**

- Students should learn the principles for designing numerical schemes for PDEs.
- Students should learn to make a connection between the mathematical equations or properties and the corresponding physical meanings.
- Students should be able to analyze the consistency, stability, and convergence of a numerical scheme.
- Students should know, for each type of PDEs (hyperbolic, parabolic, and elliptic), what kind of numerical methods are best suited for and the reasons behind these choices.
- Students should be able to use a programming language or software (Matlab or Maple) to implement and test the numerical schemes.

3. **Course Content:**

Unit-wise distribution of content and number of lectures

Unit	Topics	Sub-topic	Lectures
1	**Parabolic partial differential equations**	Explicit and implicit finite difference schemes; Truncation errors and consistency; Stability analysis: matrix method, maximum principle, Fourier analysis, energy method; maximum principle; Thomas algorithm; ADI method; general boundary conditions and nonlinear problems.	11
2	**Stability Criterion**	Consistency; convergence and stability; mesh and norms; finite difference approximations, consistency; order of accuracy and convergence; stability and the Lax equivalence theorem; calculating stability conditions.	10
3	**Hyperbolic partial differential**	Characteristics, The CFL condition, error analysis of the upwind schemes, Fourier analysis of the upwind scheme, The Lax-Wendroff scheme, The Lax-	10

	equations	Wendroff method for conservation laws.	
4	**Elliptic partial differential equations**	A model problem; Error analysis of the model problem; The general diffusion equation; boundary conditions on a curved boundary; Error analysis using a maximum principle; Asymptotic error estimates.	10
	Total		41

4. Readings

4.1 Textbook:

I. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations*, Cambridge University Press, 2nd Ed.

II. W. F. Ames, *Numerical Methods for Partial Differential Equations*, Academic Press, 3rd Ed.

4.2 Reference books:

I. Courant, R. and Hilbert, D., *Methods of Mathematical Physics, Vol-2: Partial Differential Equations*, New York, Wiley-Interscience.

5 Outcome of the Course:

After completion of the course students

- Should be able to apply numerical methods for partial differential equations.
- Have ability to determine the stability/convergence criterion for a numerical scheme.
- Would be able to apply the methods to solve physical/engineering problems.