

Tentative Syllabus for Ph. D. Admission (Mathematical Sciences)

Section 1: Calculus

Finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property; Sequences and series, convergence; Limits, continuity, uniform continuity, differentiability, mean value theorems; Riemann integration, Improper integrals; Functions of two or three variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers; Double and Triple integrals and their applications; Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

Section 2: Linear Algebra

Finite dimensional vector spaces over real or complex fields; Linear transformations and their matrix representations, rank and nullity; systems of linear equations, eigenvalues and eigenvectors, minimal polynomial, Cayley-Hamilton Theorem, diagonalization, Jordan canonical form, symmetric, skew-symmetric, Hermitian, skew-Hermitian, orthogonal and unitary matrices; Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, definite forms.

Section 3: Real Analysis

Metric spaces, connectedness, compactness, completeness; Sequences and series of functions, uniform convergence; Weierstrass approximation theorem; Power series; Functions of several variables: Differentiation, contraction mapping principle, Inverse and Implicit function theorems; Lebesgue measure, measurable functions; Lebesgue integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.

Section 4: Complex Analysis

Analytic functions, harmonic functions; Complex integration: Cauchy's integral theorem and formula; Liouville's theorem, maximum modulus principle, Morera's theorem; zeros and singularities; Power series, radius of convergence, Taylor's theorem and Laurent's theorem; residue theorem and applications for evaluating real integrals.

Section 5: Ordinary Differential equations

First order ordinary differential equations, existence and uniqueness theorems for initial value problems, linear ordinary differential equations of higher order with constant coefficients; Second order linear ordinary differential equations with variable coefficients; Cauchy-Euler equation, series solutions.

Section 6: Functional Analysis

Normed linear spaces, Banach spaces, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness; Inner-product spaces, Hilbert spaces, orthonormal bases, Riesz representation theorem.

Section 7: Numerical Analysis

Numerical solutions of algebraic and transcendental equations: bisection, secant method, Newton-Raphson method, fixed point iteration; Interpolation: error of polynomial interpolation, Lagrange and Newton interpolations; Numerical differentiation; Numerical integration: Trapezoidal and Simpson's rules; Numerical solution of a system of linear equations: direct methods (Gauss elimination, LU decomposition), iterative methods (Jacobi and Gauss-Seidel); Numerical solution of initial value problems of ODEs: Euler's method, Runge-Kutta methods of order 2.

Section 8: Partial Differential Equations

Linear and quasi-linear first order partial differential equations, method of characteristics; Second order linear equations in two variables and their classification; Cauchy, Dirichlet and Neumann problems; Solutions of Laplace and wave equations in two dimensional Cartesian coordinates, interior and exterior Dirichlet problems in polar coordinates; Separation of variables method for solving wave and diffusion equations in one space variable.

Section 9: Probability and Statistics

Random Variables (univariate & bivariate), Distributions (discrete & continuous), Characteristics of Data, Probability laws and Limit Theorems. Estimation & Hypothesis testing.